Math 318 Geometer's Sketchpad In-Class Work

Explorations

1. Make a scalene triangle (one with no sides of the same length). Make the three medians of this triangle. A median is a line segment from a vertex to the midpoint of the opposite side. Make the point at which the medians intersect. This is called the centroid of the triangle. (It is also the center of gravity or the balancing point for the triangle.) Label the centroid with the word "centroid". Hide the medians. Now drag around your triangle while you keep an eye on the centroid. Does it stay inside the triangle? Can it ever be on a side of the triangle? If so, can it be anywhere on a side?

2. Continue using the same scalene triangle. Make the three altitudes of the triangle. An altitude is a line segment from a vertex to the opposite side so that the segment hits the opposite side at a right angle. Make the point at which the three altitudes meet. This point is called the orthocenter of the triangle. Label the orthocenter with the word "orthocenter". Hide the altitudes. Now drag around your triangle while you keep an eye on the orthocenter. Does it stay inside the triangle? Can it ever be on a side of the triangle? If so, can it be anywhere on a side?

3. Continue using the same scalene triangle. Make the three perpendicular bisectors of the triangle. Make the point at which the perpendicular bisectors meet. This point is called the circumcenter. Label the circumcenter with the word "circumcenter". Hide the perpendicular bisectors. Now drag around your triangle while you keep an eye on the circumcenter. Does it stay inside the triangle? Can it ever be on a side of the triangle? If so, can it be anywhere on a side?

4. The three points you constructed can all be considered "centers" of your triangle. Now connect these three centers of the triangle with a line segment. (Are these three points collinear? That is, do they always lie on the same line?) Drag around your triangle and keep an eye on the line. Does the line ever appear to also be collinear with the midpoint of a side of the triangle? If that happens, when does it happen? What other interesting properties does this line seem to have? This line is called the Euler line, named after the famous mathematician.

5. Hide the Euler line, the centroid, and the orthocenter. Make a line segment connecting the circumcenter to any one of the triangle's vertices. Select this line segment and the circumcenter, and then, under the Construct menu, select "Circle by center and radius". What do you notice? How would you complete these sentences? "The circumcenter is the center of the circle which ______________________________." "The circumcenter of a triangle is equidistant from ______________________________."
6. Open a new sketch. Make a scalene triangle. Make the sides of this triangle some pretty color. Build an equilateral triangle onto each side of the original scalene triangle. Make the centroids of the three new triangles. Hide the lines you used to find the centroids. Connect the three centroids with line segments to make yet another triangle. What seems to be true about this new triangle? Is this always true? This discovery is credited to Napolean Bonaparte, who was a bit of a mathematician, in addition to a famous general.